CHAPTER 8

The Ultrastable System

8/1. Our problem, stated briefly at the end of Chapter 5, can now be stated finally. The type-problem was the kitten whose behaviour towards a fire was at first chaotic and unadapted, but whose behaviour later became effective and adapted. We have recognised (S. 5/8) that the property of being ‘adapted’ is equivalent to that of having the variables, both of the animal and of the environment, so co-ordinated in their actions on one another that the whole system is stable. We now know, from S. 6/3 and 7/8, that an observed system can change from one form of behaviour to another only if parameters have changed value. Since we assumed originally that no *deus ex machina* may act on it, the changes in the system must be due to step-functions acting within the whole absolute system. Our problem therefore takes the final form: *Step-functions by their changes in value are to change the behaviour of the system; what can ensure that the step-functions shall change appropriately?* The answer is provided by a principle, relating step-functions and fields, which will now be described.

8/2. In S. 7/8 it was shown that when a step-function changes value, the field of the main variables is changed. The process was illustrated in Figures 7/8/1 and 7/8/2. This is the action of step-function on field.

8/3. There is also a reciprocal action. Fields differ in the relation of their lines of behaviour to the critical states. Thus, if a representative point is started at random in the region to the left of the critical states in Figure 8/3/1, the proportion which will encounter critical states is, in I—1, in II—0, and in III—about a half. So, given a distribution of critical states and a distribution of initial states, a change of field will, in general,
change the proportion of representative points encountering critical states.

The ultrastable system

8/4. The two factors of the two preceding sections will now be found to generate a process, for each in turn evokes the other's action. The process is most clearly shown in what I shall call an ultrastable system: one that is absolute and contains step-functions in a sufficiently large number for us to be able to ignore the finiteness of the number. Consider the field of its main variables after the representative point has been released from some state. If the field leads the point to a critical state, a step-function will change value and the field will be changed. If the new field again leads the point to a critical state, again a step-function will change and again the field will be changed; and so on. The two factors, then, generate a process.

8/5. Clearly, for the process to come to an end it is necessary and sufficient that the new field should be of a form that does not lead the representative point to a critical state. (Such a field will be called terminal.) But the process may also be described in rather different words: if we watch the main variables only, we shall see field after field being rejected until one is retained: the process is selective towards fields.

As this selectivity is of the highest importance for the solution of our problem, the principle of ultrastability will be stated formally: an ultrastable system acts selectively towards the fields of the main variables, rejecting those that lead the representative point to a critical state but retaining those that do not.

This principle is the tool we have been seeking; the previous
chapters have been working towards it: the later chapters will develop it.

8/6. In the previous sections, the critical states of the step-functions were unrestricted in position; but such freedom does not correspond with what is found in biological systems (S. 9/8), so we will examine the behaviour of an ultrastable system whose critical states are so sited that they surround a definite region in the main-variables' phase-space. (At first we shall assume that the main variables are all full-functions, though the definition makes no such restriction. Later (S. 11/8) we shall examine other possibilities.)

8/7. The simplest way to demonstrate the properties of this system is by an example. Suppose there are only two main variables, $A$ and $B$, and the critical states of all the step-functions are distributed as the dots in Figure 8/7/1. Suppose the first field is that of Figure 8/7/1 (I), and that the system is started with the representative point at $X$. The line of behaviour from
X is not stable in the region, and the representative point follows the line to the boundary. Here (Y) it meets a critical state and a step-function changes value; a new field, perhaps like II, arises. The representative point is now at Y, and the line from this point is still unstable in regard to the region. The point follows the line of behaviour, meets a critical state at Z, and causes a change of a step-function; a new field (III) arises. The point is at Z, and the field includes a stable resting state, but from Z the line leads further out of the region. So another critical state is met, another step-function changes value, and a new field (IV) arises. In this field, the line of behaviour from Z is stable with regard to the region. So the representative point moves to the resting state and stops there. No further critical states are met, no further step-functions change value, and therefore no further changes of field take place. From now on, if the field of the main variables is examined, it will be found to be stable. *If the critical states surround a region, the ultrastable system is selective for fields that are stable within the region.*

(This statement is not rigorously true, for a little ingenuity can devise fields of bizarre type which are not stable but which are, under the present conditions, terminal. A fully rigorous statement would be too clumsy for use in the next few chapters; but the difficulty is only temporary, for S. 13/4 introduces some practical factors which will make the statement practically true.)

The Homeostat

8/8. So far the discussion of step-functions and of ultrastability has been purely logical. In order to provide an objective and independent test of the reasoning, a machine has been built according to the definition of the ultrastable system. This section will describe the machine and will show how its behaviour compares with the prediction of the previous section.

The homeostat (Figure 8/8/1) consists of four units, each of which carries on top a pivoted magnet (Figure 8/8/2, M in Figure 8/8/3). The angular deviations of the four magnets from the central positions provide the four main variables.

Its construction will be described in stages. Each unit emits a D.C. output proportional to the deviation of its magnet from the central position. The output is controlled in the following
Figure 8/8/1: The homeostat. Each unit carries on top a magnet and coil such as that shown in Figure 8/8/2. Of the controls on the front panel, those of the upper row control the potentiometers, those of the middle row the commutators, and those of the lower row the switches $S$ of Figure 8/8/3.

Figure 8/8/2: Typical magnet (just visible), coil, pivot, vane, and water potentiometer with electrodes at each end. The coil is quadruple, consisting of $A$, $B$, $C$ and $D$ of Figure 8/8/3.
way. In front of each magnet is a trough of water; electrodes at each end provide a potential gradient. The magnet carries a wire which dips into the water, picks up a potential depending on the position of the magnet, and sends it to the grid of the triode. \( J \) provides the anode-potential at 150 V, while \( H \) is at 180 V; so \( E \) carries a constant current. If the grid-potential allows just this current to pass through the valve, then no current will flow through the output. But if the valve passes more, or less, current than this, the output circuit will carry the difference in one direction or the other. So after \( E \) is adjusted, the output is approximately proportional to \( M \)'s deviation from its central position.

Next, the units are joined together so that each sends its output to the other three; and thereby each receives an input from each of the other three.

These inputs act on the unit's magnet through the coils \( A \), \( B \), and \( C \), so that the torque on the magnet is approximately proportional to the algebraic sum of the currents in \( A \), \( B \), and \( C \). (\( D \) also affects \( M \) as a self-feedback.) But before each input current reaches its coil, it passes through a commutator.
(X), which determines the polarity of entry to the coil, and through a potentiometer (P), which determines what fraction of the input shall reach the coil.

As soon as the system is switched on, the magnets are moved by the currents from the other units, but these movements change the currents, which modify the movements, and so on. It may be shown (S. 19/11) that if there is sufficient viscosity in the troughs, the four-variable system of the magnet-positions is approximately absolute. To this system the commutators and potentiometers act as parameters.

When these parameters are given a definite set of values, the magnets show some definite pattern of behaviour; for the parameters determine the field, and thus the lines of behaviour. If the field is stable, the four magnets move to the central position, where they actively resist any attempt to displace them. If displaced, a co-ordinated activity brings them back to the centre. Other parameter-settings may, however, give instability; in which case a 'runaway' occurs and the magnets diverge from the central positions with increasing velocity.

So far, the system of four variables has been shown to be dynamic, to have Figure 4/12/1 (A) as its diagram of immediate effects, and to be absolute. Its field depends on the thirty-two parameters X and P. It is not yet ultrastable. But the inputs, instead of being controlled by parameters set by hand, can be sent by the switches S through similar components arranged on a uniselector (or 'stepping-switch') U. The values of the components in U were deliberately randomised by taking the actual numerical values from Fisher and Yates' Table of Random Numbers. Once built on to the uniselectors, the values of these parameters are determined at any moment by the positions of the uniselectors. Twenty-five positions on each of four uniselectors (one to each unit) provide 390,625 combinations of parameter-values. In addition, the coil C of each unisector is energised when, and only when, the magnet M diverges far from the central position; for only at extreme divergence does the output-current reach a value sufficient to energise the relay F which closes the coil-circuit. A separate device, not shown, interrupts the coil-circuit regularly, making the unisector move from position to position as long as F is energised.

The system is now ultrastable; its correspondence with the
definition will be shown in each of the three requirements. Firstly, the whole system, now of eight variables (four of the magnet-deviations and four of the unisector-positions), is absolute, because the values of the eight variables are sufficient to determine its behaviour. Secondly, the variables may be divided into main variables (the four magnet-deviations), and step-functions (the variables controlled by the unisector-positions). Thirdly, as the unisectors provide an almost endless supply of step-function values (though not all different) we do not have to consider the possibility that the supply of step-function changes will come to an end. In addition, the critical states (those magnet-deviations at which the relay closes) are all situated at about a 45° deviation; so in the phase-space of the main variables they form a 'cube' around the origin.

It should be noticed that if only one, two, or three of the units are used, the resulting system is still ultrastable. It will have one, two, or three main variables respectively, but the critical states will be unaltered in position.

![Diagram](image)

**Figure 8/8/4:** Behaviour of one unit fed back into itself through a unisector. The upper line records the position of the magnet, whose side-to-side movements are recorded as up and down. The lower line (U) shows a cross-stroke whenever the unisector moves to a new position. The first movement at each D was forced by the operator, who pushed the magnet to one side to make it demonstrate the response.

Its ultrastability can now be demonstrated. First, for simplicity, is shown a single unit arranged to feed back into itself through a single unisector coil such as A, D being shorted out. In such a case the occurrence of the first negative setting on the unisector will give stability. Figure 8/8/4 shows a typical tracing. At first the step-functions gave a stable field to the single main variable, and the downward part of D₁, caused by the operator deflecting the magnet, is promptly corrected by the system, the magnet returning to its central position. At R₁,
the operator reversed the polarity of the output-input junction, making the system unstable (S. 20/7). As a result, a runaway developed, and the magnet passed the critical state (shown by the dotted line). As a result the unisector changed value. As it happened, the first new value provided a field which was stable, so the magnet returned to its central position. At \( D_2 \), a displacement showed that the system was now stable (though the return after \( R_1 \) demonstrated it too).

At \( R_2 \) the polarity of the join was reversed again. The value on the unisector was now no longer suitable, the field was unstable, and a runaway occurred. This time three unisector positions provided three fields which were all unstable: all were

\[
\begin{array}{c}
1 \\
R_1 \\
2 \\
D_1 \\
D_2 \\
D_3 \\
\text{Time} \\
\end{array}
\]

Figure 8/8/5: Two units (1 and 2) interacting. (Details as in Fig. 8/8/4.)

rejected. But the fourth was stable, the magnet returned to the centre, no further unisector changes occurred, and the single main variable had a stable field. At \( D_3 \) its stability was again demonstrated.

Figure 8/8/5 shows another experiment, this time with two units interacting. The diagram of immediate effects was \( 1 \to 2 \); the effect \( 1 \to 2 \) was hand-controlled, and \( 2 \to 1 \) was unisector-controlled. At first the step-function values combined to give stability, shown by the responses to \( D_1 \). At \( R_1 \), reversal of the commutator by hand rendered the system unstable, a runaway occurred, and the variables transgressed the critical states. The unisector in Unit 1 changed position and, as it happened, gave at its first trial a stable field. It will be noticed that whereas before \( R_1 \) the upstroke of \( D_1 \) in 2 caused an upstroke in 1, it
caused a downstroke in 1 after $R_2$, showing that the action $2 \rightarrow 1$ had been reversed by the unisector. This reversal compensated for the reversal of $1 \rightarrow 2$ caused at $R_1$.

At $R_2$ the whole process was repeated. This time three unisector changes were required before stability was restored. A comparison of the effect of $D_2$ on 1 with that of $D_3$ shows that compensation has occurred again.

The homeostat can thus demonstrate the elementary facts of ultrastability.

8/9. In what way does an ultrastable system differ from an ordinary stable system?

In one sense the two systems are similar. Each is assumed absolute, and if therefore we form the field of all its variables, each will have one permanent field. Given a region, every line of behaviour is permanently stable or unstable (see Figure 7/8/1). Viewed in this way, the two systems show no essential difference. But if we compare the variables of the stable system with only the main variables of the ultrastable, then an obvious difference appears: the field of the stable system is single and permanent, but in the ultrastable system the phase-space of the main variables shows a succession of transient fields concluded by a terminal field which is always stable. The distinction in actual behaviour can best be shown by an example. The automatic pilot is a device which, amongst other actions, keeps the aeroplane horizontal. It must therefore be connected to the ailerons in such a way that when the plane rolls to the right, its output must act on them so as to roll the plane to the left. If properly joined, the whole system is stable and self-correcting: it can now fly safely through turbulent air, for though it will roll frequently, it will always come back to the level. The homeostat, if joined in this way, would tend to do the same. (Though not well suited, it would, in principle, if given a gyroscope, be able to correct roll.)

So far they show no difference; but connect the ailerons in reverse and compare them. The automatic pilot would act, after a small disturbance, to increase the roll, and would persist in its wrong action to the very end. The homeostat, however, would persist in its wrong action only until the increasing deviation made the step-functions start changing. On the occurrence
of the first suitable new value, the homeostat would act to stabilise instead of to overthrow; it would return the plane to the horizontal; and it would then be ordinarily self-correcting for disturbances.

There is therefore some justification for the name 'ultrastable'; for if the main variables are assembled so as to make their field unstable, the ultrastable system will change this field till it is stable. The degree of stability shown is therefore of an order higher than that of the system with a single field.

Another difference can be seen by considering the number of factors which need adjustment or specification in order to achieve stability. Less adjustment is needed if the system is ultrastable. Thus an automatic pilot must be joined to the ailerons with care, but an ultrastable pilot could safely be joined to the ailerons at random. Again, a linear system of $n$ variables, to be made stable, needs the simultaneous adjustment of at least $n$ parameters (S. 20/11, Ex. 3). If $n$ is, say, a thousand, then at least a thousand parameters must be correctly adjusted if stability is to be achieved. But an ultrastable system with a thousand main variables needs, to achieve stability, the specification of about six factors; for this is approximately the number of independent items in the specification of the system (S. 9/9). A large system, then, can be made stable with much less detailed specification if it is made ultrastable.

8/10. In S. 6/2 it was shown that every dynamic system is acted on by an indefinitely large number of parameters, many of which are taken for granted, for they are always given well-understood 'obvious' values. Thus, in mechanical systems it is taken for granted, unless specially mentioned, that the bodies carry a zero electrostatic charge; in physiological experiments, that the tissues, unless specially mentioned, contain no unusual drug; in biological experiments, that the animal, unless specially mentioned, is in good health. All these parameters, however, are effective in that, had their values been different, the variables would not have followed the same line of behaviour. Clearly the field of an absolute system depends not only on those parameters which have been fixed individually and specifically, but on all the great number which have been fixed incidentally.

Now the ultrastable system proceeds to a terminal field which
is stable in conjunction with all the system’s parameter-values (and it is clear by the principle of ultrastability that this must be so, for whether the parameters are at their ‘usual’ values or not is irrelevant). The ultrastable system will therefore always produce a set of step-function values which is so related to the particular set of parameter-values that, in conjunction with them, the system is stable. If the parameters have unusual values,

![Diagram showing step-functions](image)

Figure 8/10/1: Three units interacting. At J, units 1 and 2 were constrained to move together. New step-function values were found which produced stability. These values give stability in conjunction with the constraint, for when it is removed, at R, the system becomes unstable.

the step-functions will also finish with values that are compensatingly unusual. To the casual observer this adjustment of the step-function values to the parameter-values may be surprising; we, however, can see that it is inevitable.

The fact is demonstrable on the homeostat. After the machine was completed, some ‘unusual’ complications were imposed on it (‘unusual’ in the sense that they were not thought of till the machine had been built), and the machine was then tested to see how it would succeed in finding a stable field when affected by the peculiar complications. One such test was
made by joining the front two magnets by a light glass fibre so that they had to move together. Figure 8/10/1 shows a typical record of the changes. Three units were joined together and were at first stable, as shown by the response when the operator displaced magnet 1 at $D_1$. At $J$, the magnets of 1 and 2 were joined so that they could move only together. The result of the constraint in this case was to make the system unstable. But the instability evoked step-function changes, and a new terminal field was found. This was, of course, stable, as was shown by its response to the displacement, made by the operator, at $D_2$. But it should be noticed that the new set of step-function values was adjusted to, or 'took notice of', the constraint and, in fact, used it in the maintenance of stability; for when, at $R$, the operator gently lifted the fibre away the system became unstable.

References


